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Role of thermally stratified medium on a free convection flow from a rotating sphere

P. Saikrishnan
Centre for Differential Equations, Continuum Mechanics and Applications, School of Computational and Applied Mathematics, University of the Witwatersrand, Johannesburg, South Africa

Satyajit Roy
Department of Mathematics, Indian Institute of Technology Madras, Chennai, India

H.S. Takhar
Department of Engineering, Manchester Metropolitan University, Manchester, UK, and

R. Ravindran
Centre for Differential Equations, Continuum Mechanics and Applications, School of Computational and Applied Mathematics, University of the Witwatersrand, Johannesburg, South Africa

Abstract

Purpose – The purpose of this paper is to study the influence of thermally stratified medium on a free convection flow from a sphere, which is rotating about the vertical axis, immersed in a stably thermally stratified medium.

Design/methodology/approach – An implicit finite-difference scheme in combination with the quasi-linearization technique is applied to obtain the steady state non-similar solutions of the governing boundary layer equations for flow and temperature fields.

Findings – The numerical results indicate that the heat transfer rate at the wall decreases significantly with an increasing thermal stratification parameter, but its effect on the skin friction coefficients is rather minimum. In fact, the presence of thermal stratification of the medium influences the heat transfer at wall to be in opposite direction, that is, from fluids to the wall above a certain height. The heat transfer rate increases but the skin frictions decrease with the increase of Prandtl number. In particular, the effect of buoyancy force is much more sensitive for low Prandtl number fluids ($Pr = 0.7$, air) than that of high Prandtl number fluids ($Pr = 7$, water). Also the skin friction in rotating direction is less sensitive to the buoyancy force as the buoyancy force acts in the streamwise direction for the present study of thermally stratified medium.

Research limitations/implications – The ambient temperature T_∞ is assumed to increase linearly with height h. The viscous dissipation term, which is usually small for natural convection flows, has been neglected in the energy equation. The flow is assumed to be axi-symmetric. The Boussinesq approximation is invoked for the fluid properties to relate density changes to temperature changes, and to couple in this way the temperature field to the flow field.

Practical implications – Free convection in a thermally stratified medium occurs in many environmental processes with temperature stratification, and in industrial applications within a closed chamber with heated walls. Also, free convections associated with heat rejection systems for long-duration deep ocean powder modules where ocean environment is stratified are examples of such type.

Originality/value – The research presented in this paper investigates the free convection flow on a sphere, which is rotating with a constant angular velocity along its vertical axis in a stably thermally stratified fluid.

Keywords Convection, Flow, Heat transfer media, Temperature distribution

Paper type Research paper

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Nomenclature

a	constant
C_{fx}	local skin friction coefficient in streamwise direction
$C_{f\phi}$	local skin friction coefficient in rotation direction
f	dimensionless stream function
f'	dimensionless velocity along the x-direction
g	acceleration due to gravity, m s^{-2}
Gr	Grashof number
h	height, m
n	constant
Nu	Nusselt number
Pr	Prandtl number
r	radial distance, m
R	radius of the sphere, m
Re_R	Reynolds number
s	dimensionless velocity along ϕ
s_o	stratification parameter
T	temperature, K
u, v, w	velocity components, m s^{-1}
U	characteristic velocity, m s^{-1}
x, ϕ, z	curvilinear coordinates

Greek letters

α	thermal diffusivity, m^2 s^{-1}
β	angle (x/R)
$\bar{\beta}$	volumetric coefficient of thermal expansion, K^{-1}
η, ξ	transformed similarity variables
μ	dynamic viscosity, kg m^{-1} s^{-1}
ν	kinematic viscosity, m^2 s^{-1}
ρ	density, kg m^{-3}
Ω	angular velocity
λ	buoyancy parameter
ψ	stream function, m^2 s^{-1}
θ	dimensionless temperature
ϕ	circumferential angle

Subscripts

w	condition at the wall
∞	condition at infinity
x, z	denote partial derivatives with respect to these variables

1. Introduction

Free convection in a thermally stratified medium occurs in many environmental processes with temperature stratification, and in industrial applications within a closed chamber with heated walls. Also, free convections associated with heat rejection systems for long-duration deep ocean powder modules where ocean environment is stratified are examples of such type. Early works by Jaluria and Gebhart (1974), Chen and Eichhron (1976) and Henky and Hoogendoorn (1989) have considered the free convection flows over a vertical flat surface embedded in a thermally stratified fluid. Angirasa and Srinivasan (1989) presented a numerical study of the double diffusive free convection flow adjacent to a vertical surface in a thermally stratified ambient medium due to the combined effects of buoyancy forces caused by heat and mass diffusion. In their analysis, they studied the role of ambient thermal stratification by considering the simple case of linear temperature variations. Singh (1977) studied the free convection flow from a fixed sphere in a slightly thermally stratified fluid. The natural convection from a fixed cylinder and sphere immersed in a thermally stratified fluid was considered by Chen and Eichhorn (1979). On the other hand, Eichhorn *et al.* (1974) studied the above problem experimentally.

The effects of buoyancy force on the flow and heat transfer at the forward stagnation point of a sphere rotating in an ambient fluid were considered by Suwono

(1980). Also Sparrow and Cess (1962) have investigated the flow over an infinite rotating disk including the effects of magnetic field and heat transfer. In both the studies, self-similar solution were obtained numerically. In a recent study, Hossain *et al.* (2002) considered the natural convection flow along a vertical circular cone with uniform surface temperature and surface heat flux in a thermally stratified medium. They have investigated the effect of stratification on free convection from the vertical cone by taking the ambient temperature as linear function of the distance measured from the apex of the cone. Furthermore, in recent past, various studies on natural convection flow including the effect of thermal stratification of medium are reported in the literature by Abdulkarim and Jaluria (1996), Raghavarao and Srinivas (1998), Al-Najem *et al.* (1998), Hasnaoui *et al.* (1993), Shih-Wen Hsiao (1998), Swarnendu and Sarkar (1995), Kiwan and Zeitoun (2008), Singh and Tiwari (1993) and Takhar *et al.* (2001).

The aim of the present study is to investigate the free convection flow on a sphere, which is rotating with a constant angular velocity along its vertical axis in a stably thermally stratified fluid. The present analysis is an extension of the work by Singh (1977) and Chen and Eichhorn (1979) to include the effect of the rotation of the body or of Suwono (1980) to include the effect of thermal stratification. However, our results are more accurate than those of Singh (1977) and Chen and Eichhorn (1979), where perturbation methods were used. Also, the results are more general than those of Suwono (1980) who considered the stagnation flow. The coupled non-linear parabolic partial differential equations governing the flow have been solved numerically by using an implicit finite-difference scheme in combination with the quasi-linearization technique Inouye and Tate (1974) and Ravindran *et al.* (2008). The present results have been compared with those of Chen and Eichhorn (1979), Eichhorn *et al.* (1974), Suwono (1980) and Sparrow and Cess (1962).

2. Formulation of the problem

Let us consider a heated sphere with a constant wall temperature T_w rotating with a constant angular velocity Ω about its vertical axis in a stably thermally stratified ambient fluid. The ambient temperature T_∞ is assumed to increase linearly with height h. The buoyancy force arises due to the temperature difference in the fluid. The viscous dissipation term, which is usually small for natural convection flows, has been neglected in the energy equation. We choose a non-rotating orthogonal curvilinear coordinate system (x, ϕ, z) as shown in Figure 1, where x is the distance measured along a meridian curve from the forward stagnation point, ϕ is the circumferential angle and z is the distance normal to the surface of the rotating sphere. The quantity $r(x)$ is the radial distance from a surface point x to the axis of rotation. R is the radius of the sphere. u, v and w are the velocity components along x-, ϕ- and z- directions, respectively. The gravity force g acts parallel to the axis of rotation. The flow is assumed to be axi-symmetric. Hence, the velocity and temperature fields are independent of the angle ϕ. The Boussinesq approximation is invoked for the fluid properties to relate density changes to temperature changes, and to couple in this way the temperature field to the flow field. Under the foregoing assumptions, the boundary layer equations based on the conservation of mass, momentum and energy governing the steady laminar natural convection flow over a rotating sphere in a thermally stratified medium can be expressed as (Chen and Eichhorn, 1979; Suwono, 1980; Schlichting and Gersten, 2000):

$$(ru)_x + (rw)_z = 0 \tag{1}$$

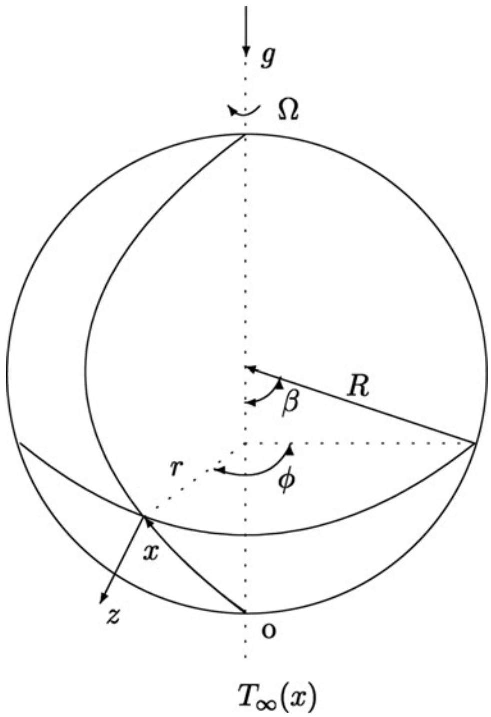

Figure 1.
Flow model and coordinate system

$$uu_x + wu_z - \left(\frac{v^2}{r}\right) r_x = \nu u_{zz} + g\bar{\beta}(T - T_\infty(x)) \sin\beta, \tag{2}$$

$$uv_x + wv_z + \frac{uv}{r} r_x = v_{zz} \tag{3}$$

$$uT_x + wT_z = \alpha T_{zz} \tag{4}$$

The suitable boundary conditions are:

$$\begin{aligned} &u(x,0), \quad v(x,0) = \Omega r, \quad w(x,0) = 0, \quad T(x,0) = T_w, \\ &u(x,\infty) = 0, \quad v(x,\infty) = 0, \quad T(x,\infty) = T_\infty(x), \\ &u(0,z) = v(0,z) = 0, \quad T(0,z) = T_o, z > 0 \end{aligned} \tag{5}$$

Here T is the temperature; ν is the kinematics viscosity; $\bar{\beta}$ is the volumetric coefficient of thermal expansion; α is the thermal diffusivity; T_w is the uniform wall temperature; $T_\infty(x)(= T_o + ah)$ is the ambient temperature; a is the positive constant; $h(= R(1 - \cos\beta))$ is the height; R is the radius of the sphere; $\beta(= x/R$ is the angle (see Figure 1) and T_o is the value of the ambient temperature $T_\infty(x)$ at $x = 0(\beta = 0)$.

It is convenient to transform equations (1)-(4) into a dimensionless form. Hence, we apply the following transformations to equations (1)-(4):

$$
\begin{aligned}
&\xi=\int_o^x \sin\beta\left(\frac{r}{R}\right)^2 d\left(\frac{x}{R}\right), \quad \eta=\left(\frac{Re_R}{2\xi}\right)^{1/2}\sin\beta\left(\frac{r}{R}\right)\left(\frac{z}{R}\right),\\
&r=R\sin\beta, \quad U=\Omega R, \quad \beta=\frac{x}{R}, \quad \psi(x,z)=UR^2\left(\frac{2\xi}{Re_R}\right)^{1/2} f(\xi,\eta),\\
&ru=\left(\frac{\partial\psi}{\partial z}\right), \quad rw=-\left(\frac{\partial\psi}{\partial x}\right), \quad u=\Omega r f'(\xi,\eta), \quad v=\Omega r s(\xi,\eta),\\
&Re_R\frac{UR}{\nu}, \quad \lambda=\frac{Gr_R}{Re_R^2}, \quad Pr=\frac{\nu}{\alpha}, \quad Gr_R=g\bar{\beta}(T_w-T_o)R^3/\nu^2,\\
&T-T_\infty=(T_w-T_o)\theta(\xi,\eta), \;\; s_o=\frac{aR}{T_w-T_o}, a>0, \;\; T_\infty=T_o+aR(1-\cos\beta).
\end{aligned} \tag{6}
$$

Consequently, equation (1) is identically satisfied and equations (2)-(4) reduce to:

$$
f'''+ff''-P_1(\beta)f'^2+P_2(\beta)s^2+P_3(\beta)\lambda\theta=P_4(\beta)\left[f'\frac{\partial f'}{\partial\beta}-f''\frac{\partial f}{\partial\beta}\right], \tag{7}
$$

$$
s''+fs'-P_5(\beta)f's=P_4(\beta)\left[f'\frac{\partial s}{\partial\beta}-s'\frac{\partial f}{\partial\beta}\right], \tag{8}
$$

$$
Pr^{-1}\theta''+f\theta'-P_6(\beta)s_o f'=P_4(\beta)\left[f'\frac{\partial\theta}{\partial\beta}-\theta'\frac{\partial f}{\partial\beta}\right], \tag{9}
$$

where

$$
\begin{aligned}
&\xi=3^{-1}(1-\cos\beta)^2(2+\cos\beta), \quad 2\xi\left(\frac{\partial}{\partial\xi}\right)\equiv P_4(\beta)\left(\frac{\partial}{\partial\xi}\right),\\
&P_1(\beta)=\frac{2\xi(\frac{dr}{dx})(\frac{\Omega R}{U})}{(\frac{\Omega r}{U})^2(\frac{r}{R})^2}=\frac{2\cos\beta(2+\cos\beta)}{3(1+\cos\beta)^2},\\
&P_2(\beta)=\frac{\frac{2\xi}{r}(\frac{dr}{dx})U}{\Omega\sin\beta(\frac{r}{R})^2}=\frac{2\cos\beta(2+\cos\beta)}{3(1+\cos\beta)^2},\\
&P_3(\beta)=\frac{2\xi UR\sin\beta}{(\Omega r)^3(\frac{r}{R})^2}=\frac{2(2+\cos\beta)}{3(1+\cos\beta)^2},\\
&P_4(\beta)=2\xi\left(\frac{\partial\beta}{\partial\xi}\right)=\frac{2\sin(\beta/2)(2+\cos\beta)}{3\cos(\beta/2)(1+\cos\beta)},\\
&P_5(\beta)=\frac{4\Omega R\frac{dr}{dx}\xi}{(\frac{\Omega r}{U})^2(\frac{r}{R})^2}=\frac{4\cos\beta(2+\cos\beta)}{3(1+\cos\beta)^2},\\
&P_6(\beta)=2\xi\sin\beta\left(\frac{\partial\xi}{\partial\beta}\right)=\frac{2(1-\cos\beta)(2+\cos\beta)}{3(1+\cos\beta)}.
\end{aligned} \tag{10}
$$

The boundary conditions (5) can be re-written as:

$$\begin{aligned} &f(\beta,0) = f'(\beta,0) = 0, \quad s(\beta,0) = 1, \quad \theta(\beta,0) = 1 - s_o(1 - \cos\beta), \\ &f'(\beta,\infty) = s(\beta,\infty) = \theta(\beta,\infty) = 0. \end{aligned} \tag{11}$$

Here ξ and η are the transformed coordinates, ψ and f are the dimensional and dimensionless stream functions, respectively; θ is the dimensionless temperature; f' and s are the dimensionless velocity components along x (streamwise) and ϕ (rotating) directions, respectively; $P_i (i = 1, 2, \ldots, 6)$ are functions of the angle $\beta = (x/R)$; Gr_R is the Grashof number defined with respect to R; Re_R is the Reynolds number defined with respect to R; Pr is the Prandtl number; λ is the buoyancy parameter which is the ratio of the Grashof number to the Reynolds number squared; s_o is the thermal stratification parameter; U is the characteristic velocity and $(\,)'$ prime denotes derivative with respect to η.

The quantities of physical interest are given by:

$$\begin{aligned} C_{fx} &= \frac{2\mu(\frac{\partial u}{\partial z})_{z=0}}{\rho(\Omega R)^2} = 2(6)^{1/2}(Re_R)^{-1/2} \sin\beta\cos^2(\beta/2)(2+\cos\beta)^{-1/2} f''(\beta,0) \\ C_{f\phi} &= -\frac{2\mu(\frac{\partial v}{\partial z})_{z=0}}{\rho(\Omega R)^2} = 2(6)^{1/2}(Re_R)^{-1/2} \sin\beta\cos^2(\beta/2)(2+\cos\beta)^{-1/2} s'(\beta,0) \\ Nu &= -\frac{R(\frac{\partial T}{\partial z})_{z=0}}{T_w - T_\infty} = -6^{1/2}(Re_R)^{1/2}\cos^2(\beta/2)(2+\cos\beta)^{-1/2} \\ &\quad \times [1 - s_o(1 - cos\beta)]^{-1}\theta'(\beta,0) \end{aligned}$$

where C_{fx} and $C_{f\phi}$ are the skin friction coefficients in the streamwise and rotating directions, respectively, Nu is the Nusselt number, and ρ and μ are the density and viscosity, respectively.

It may be remarked that for $\beta = 0(\xi = 0)$, equations (7)-(9) reduce to ordinary differential equations representing the self-similar flow in the forward stagnation-point region. The self-similar equations are given by:

$$f''' + ff'' - 2^{-1}(f'^2 - s^2 - \lambda\theta) = 0 \tag{12}$$

$$s'' + fs' - f's = 0 \tag{13}$$

$$Pr^{-1}\theta'' + f\theta' = 0 \tag{14}$$

with boundary conditions:

$$\begin{aligned} &f(0) = f'(0) = 0, \quad (0) = 1, \quad \theta(0) = 1; \\ &f'(\infty) = s(\infty) = \theta(\infty) = 0. \end{aligned} \tag{15}$$

Equations (12)-(15) are identical to those of Suwono (1980) who considered the natural convectoin flow over a rotating sphere. Also, for $\lambda = 0$ (in the absence of buoyancy force), equations (12)-(15) reduce to those of Sparrow and Cess (1962) who investigated the flow and heat transfer over an infinite disk rotating in an otherwise ambient fluid if the following transformations are applied to the dependent and independent variables;

$$\eta_1 = 2^{-1/2}\eta, \quad f(\eta) = -2^{-1/2}F(\eta_1), \quad s(\eta) = S(\eta_1), \quad \theta(\eta) = \phi(\eta_1) \tag{16}$$

The above transformations imply that for direct comparison with those of Sparrow and Cess (1962), we have to multiply our results for $f'(0)$ by $-2^{3/2}$, and $s'(0)$ and $\theta'(0)$ by $2^{1/2}$, respectively.

3. Method of solution

The non-linear coupled partial differential equations (7)-(9) under boundary conditions given by equation (11) have been solved numerically using an implicit finite-difference scheme in combination with the quasi-linearization technique. An iterative sequence of linear equations is carefully constructed to approximate the non-linear equations (7)-(9) for achieving quadratic convergence and monotonicity. Applying the quasi-linearization technique (Inouye and Tate, 1974; Ravindran *et al.* 2008), we replace the non-linear partial differential equations (7)-(9) by an iterative sequence of linear equations as follows:

$$X_1^k F_{\eta\eta}^{k+1} + X_2^k F_{\eta}^{k+1} + X_3^k F^{k+1} + X_4^k F_{\beta}^{k+1} + X_5^k s_{\eta}^{k+1} + X_6^k \theta^{k+1} = X_7^k \tag{17}$$

$$Y_1^k s_{\eta\eta}^{k+1} + Y_2^k s_{\eta}^{k+1} + Y_3^k s^{k+1} + Y_4^k s_{\beta}^{k+1} + Y_5^k F^{k+1} = Y_6^k \tag{18}$$

$$Z_1^k \theta_{\eta\eta}^{k+1} + Z_2^k \theta_{\eta}^{k+1} + Z_3^k \theta^{k+1} + Z_4^k \theta_{\beta}^{k+1} + Z_5^k F^{k+1} = Z_6^k \tag{19}$$

where the coefficient functions with iterative index k are known and functions with iterative index $k = 1$ are to be determined.

The boundary conditions become:

$$\begin{aligned} &F^{k+1}(\beta,0) = 0, \quad s^{k+1}(\beta,0) = 1, \quad G^{k+1}(\beta,0) = 1 - s_o(1-\cos\beta) \\ &F^{k+1}(\beta,\eta_\infty) = 0, \quad s^{k+1}(\beta,\eta_\infty) = 0, \quad G^{k+1}(\beta,\eta_\infty) = 0 \end{aligned}$$

where $f(\xi,\eta) = \int_0^\eta F(\xi,\eta)d\eta + f(\xi,0)$ and η_∞ is the edge of the boundary layer. The coefficients in equations (17)-(19) are given by:

$$\begin{aligned} X_1^k &= 1 \\ X_2^k &= f + P_4(\beta)\frac{\partial f}{\partial \beta} \\ X_3^k &= -2P_1(\beta)F - P_4(\beta)\frac{\partial F}{\partial \beta} \end{aligned}$$

$$X_4^k = -P_4(\beta)F$$
$$X_5^k = 2P_2(\beta)s$$
$$X_6^k = P_3(\beta)\lambda$$
$$X_7^k = -P_1(\beta)F^2 + P_2(\beta)s^2 - P_4(\beta)F\frac{\partial F}{\partial \beta}$$
$$Y_1^k = 1$$
$$Y_2^k = f + P_4(\beta)\frac{\partial f}{\partial \beta}$$
$$Y_3^k = -P_5(\beta)F$$
$$Y_4^k = -P_4(\beta)F$$
$$Y_5^k = -P_5(\beta)s - P_4(\beta)\frac{\partial s}{\partial \beta}$$
$$Y_6^k = -P_5(\beta)sF - P_4(\beta)\frac{\partial s}{\partial \beta}F$$
$$Z_1^k = Pr^{-1}$$
$$Z_2^k = f + P_4(\beta)\frac{\partial f}{\partial \beta}$$
$$Z_3^k = 0$$
$$Z_4^k = -P_4(\beta)F$$
$$Z_5^k = -P_6(\beta)s_o - P_4(\beta)\frac{\partial \theta}{\partial \beta}$$
$$Z_6^k = -P_4(\beta)F\frac{\partial \theta}{\partial \beta}$$

Now, the resulting sequence of linear partial differential equations (17)-(19) were expressed in difference form using central difference scheme in η-direction and backward difference scheme in β-direction. Since the method is described for ordinary differential equations by Inouye and Tate (1974) and also explained for partial differential equations in a recent study by Ravindran *et al.* (2008), its detailed description is not provided for the sake of brevity. In each iteration step, the equations were then reduced to a system of linear algebraic equations with a block tri-diagonal structure which is solved using Varga's algorithm (Varga, 2000). The step sizes $\triangle\eta = 0.025$, $\triangle\beta = 0.025$ and the edge of the boundary layer at $\eta_\infty = 7$ were chosen after carrying out the sensitivity analysis. It is found that further reduction in $\triangle\eta$ or $\triangle\beta$ or η_∞ or in all does not change the results up to the fourth decimal place. A convergence criterion based on the relative difference between the current and previous iterations has been used. When this difference becomes 10^{-5}, the solution is assumed to have converged and the iterative process is terminated, i.e.

$$\mathrm{Max}\left\{ \left|(F_\eta)_w^{k+1} - (F_\eta)_w^k\right|, \left|(s_\eta)_w^{k+1} - (s_\eta)_w^k\right|, \left|(\theta_\eta)_w^{k+1} - (\theta_\eta)_w^k\right| \right\} < 10^{-5}.$$

4. Results and discussions

Equations (7)-(9) with boundary conditions (10) have been solved by using the implicit finite-difference scheme with the quasi-linearization technique as described in the previous section. In order to check the accuracy of the present numerical approach, comparisons of the skin friction and heat transfer results $(f''(0), -s'(0), -\theta'(0))$ for $s_o = \lambda = 0$ (without stratification and buoyancy force) and $\beta = 0$ (at the forward stagnation point), $Pr = 1$ with those of Sparrow and Cess (1962) are made and found to be in good agreement. Further, the comparisons of skin friction and heat transfer results $(f''(0), -s'(0), -\theta'(0))$ for several values of the buoyancy parameter λ when $s_o = \beta = 0$, $Pr = 0.72$ with those of Suwono (1980) are made and some of the comparisons are shown in Figure 2.

Figures 3-5 present the effects of the buoyancy parameter λ and Prandtl number Pr on the velocity and temperature profiles $(f'(\beta,\eta), s(\beta,\eta), \theta(\beta,\eta))$ for $s_o = 0.1$ at $\beta = 1.0$. Since the positive buoyancy force ($\lambda > 0$) acts like a favourable pressure gradient, the fluids within the boundary layer are accelerated resulting a thinner momentum boundary layer. Moreover, the magnitude of the maximum value for the streamwise velocity component $f'(\beta,\eta)$ increases with λ and its location shift towards the wall. The action of the buoyancy force shows that the magnitude of the maximum streamwise velocity (f') within the boundary is significantly more for lower Prandtl number fluid ($Pr = 0.7$) as compared to the higher Prandtl number fluid ($Pr = 7.0$) (see Figure 3). The reason is that the buoyancy force (λ) effect is larger in a low Prandtl number fluid (Pr = 0.7, air) due to low viscosity of the fluid which enhances the velocity within the boundary layer as the assisting bouyancy force acts like a favourable pressure gradient. But for higher Prandtl number fluid ($Pr = 7.0$, water), the magnitude of maximum streamwise velocity is less because higher Prandtl number fluid implies more viscous fluid which makes it less sensitive to the buoyancy parameter λ. In contrast, the increase in the magnitude of buoyancy forces, i.e. the increase in λ reduces the velocity component in the rotating direction $s(\beta,\eta)$ within the boundary layer for lower Prandtl number ($Pr = 0.7$). The physical reason is that

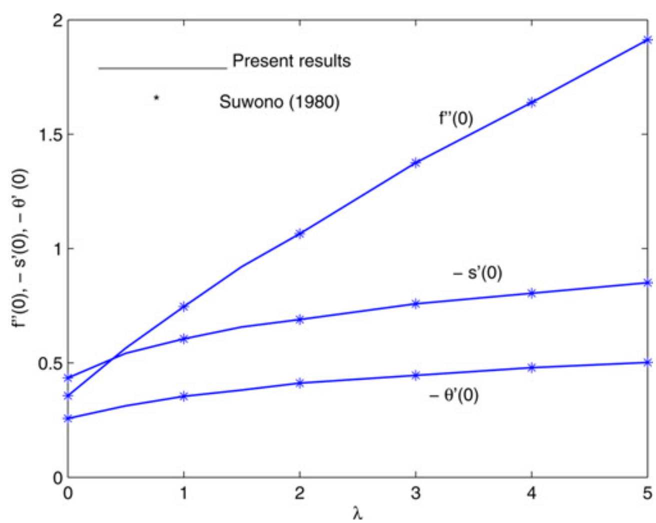

Figure 2.
Comparison of the surface shear stresses and heat transfer $(f''(0), -s'(0), -\theta'(0))$ for $s_o = \beta = 0$ and $Pr = 0.72$

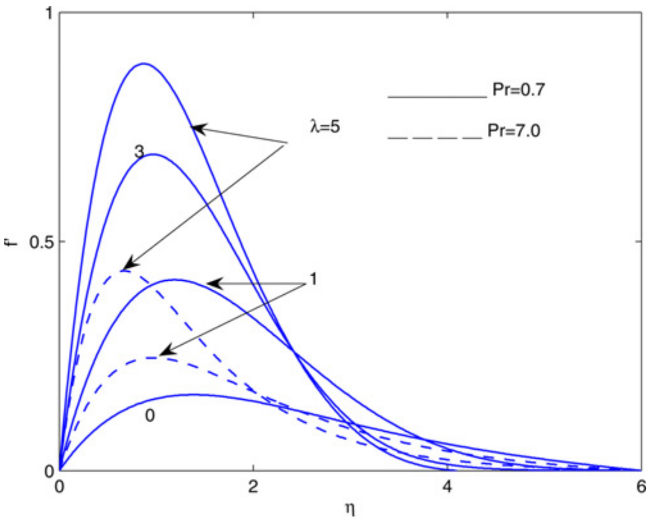

Figure 3.
Effects of λ and Pr on the velocity profile in streamwise direction $f'(\beta, \eta)$ for $s_o = 0.1$ at $\beta = 1.0$

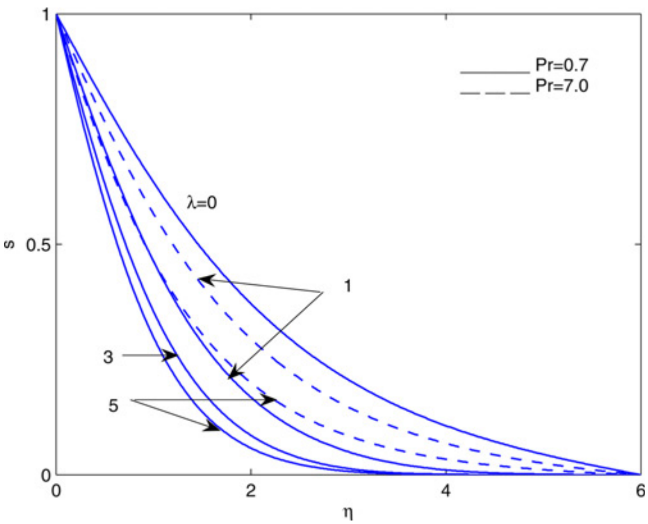

Figure 4.
Effects of λ and Pr on the velocity profile in rotating direction $s(\beta, \eta)$ for $s_o = 0.1$ at $\beta = 1.0$

the buoyancy force acts in the streamwise direction for the present problem and the increase in the magnitude of buoyancy force drags more fluids in the streamwise direction resulting a reduction in the magnitude of rotating velocity component. Furthermore, since the buoyancy force is less sensitive for higher Prandtl number fluid ($Pr = 7.0$), Figure 4 shows that the velocity component in the rotating direction ($s(\beta, \eta)$) decreases its magnitude comparatively less with the increase of λ than that for lower Prandtl number fluid ($Pr = 0.7$, air). Since the fluid gets accelerated with the increase of positive buoyancy force, it can be seen in Figure 5 that the thickness of the thermal boundary layer reduces and the temperature $\theta(\beta, \eta)$ within the boundary layer decreases with the increase of λ. It may also be noted in Figure 5 that due to thermal

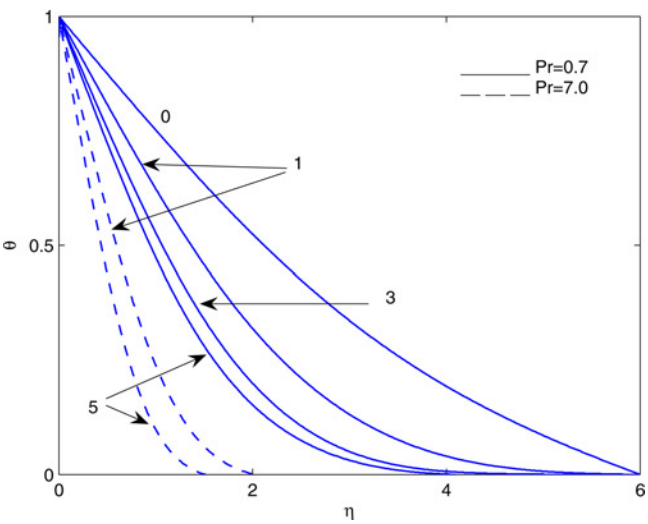

Figure 5.
Effects of λ and Pr on the temperature profile $\theta(\beta, \eta)$ for $s_o = 0.1$ at $\beta = 1.0$

stratification the non-dimensional temperature $\theta(\beta, 0)$ at the wall is slightly less to the value of $\theta(\beta, \eta), 0 < \eta \ll 1$, i.e. very close to the wall, which indicates that the heat transfer at the wall occurs in opposite direction, that is, from fluids to the wall. Since the change in temperature occurs close to the wall within a very narrow region, it is difficult to visualize in Figure 5 but it is possible to see the negative heat transfer in later Figure 8. This phenomena is a characteristic feature of thermal stratification of the medium and the effect is pronounced above a certain height, i.e. after some increase in the angle β in the presence of thermal stratification ($s_o \neq 0$). The velocity and temperature profiles $(f'(\beta, \eta), s(\beta, \eta), \theta(\beta, \eta))$ at a distant streamwise locations (i.e. at higher values of β) are comparatively less steeper than those at the previous streamwise locations (i.e. $\beta < \pi/2$). Also, the velocity and temperature profiles $(f'(\beta, \eta), s(\beta, \eta), \theta(\beta, \eta))$ near the separation point have larger η_∞ than those at the previous streamwise locations (i.e. $\beta < \pi/2$). Moreover Figure 5 shows that the increase of Prandtl number Pr results into thinner thermal boundary layer as the higher Prandtl number fluid has a lower thermal conductivity.

Figures 6-8 display the effect of buoyancy parameter λ on the local skin friction coefficients in the streamwise and rotating directions $(Re_R^{1/2} C_{fx}, Re_R^{1/2} C_{f\phi})$ and the local Nusselt number $(Re_R^{-1/2} Nu)$ for $s_o = 0.1, Pr = 0.7, 0 \leq \beta \leq 2.25$. As mentioned earlier, the positive buoyancy force ($\lambda > 0$) accelerates the fluid motion in the streamwise direction. Hence, the skin friction coefficient in the x-direction $(Re_R^{1/2} C_{fx})$ increases significantly with the increase of buoyancy force (λ). In contrast, the increase in the skin friction coefficient in rotating direction $(Re_R^{1/2} C_{f\phi})$ with the increase of λ is comparatively less and after certain increase of height (i.e. with the increase in β), $(Re_R^{1/2} C_{f\phi})$ vanishes for all values of λ. In particular, $Re_R^{1/2} C_{f\phi}$ vanishes for $\lambda = 0$ at $\beta = 1.65$ and for $\lambda \geq 1$ at $\beta = 2.21$. The physical reason is that the skin friction in rotating direction $(Re_R^{1/2} C_{f\phi})$ is less sensitive to the buoyancy force as buoyancy force acts in the x-direction in the present case of the thermally stratified medium. The skin friction coefficient in the streamwise and rotating directions $(Re_R^{1/2} C_{fx}, Re_R^{1/2} C_{f\phi})$ for a fixed λ first increases with the increase in the angle β (i.e. with the increase of height) and attains a maximum value, and then decrease. The location of the maximum value

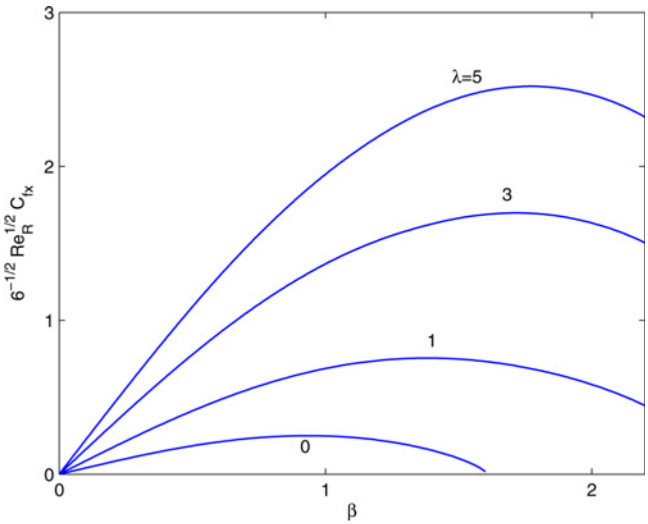

Figure 6.
Effect of λ on the skin friction coefficients in the streamwise direction for $s_o = 0.1$ and $Pr = 0.7$

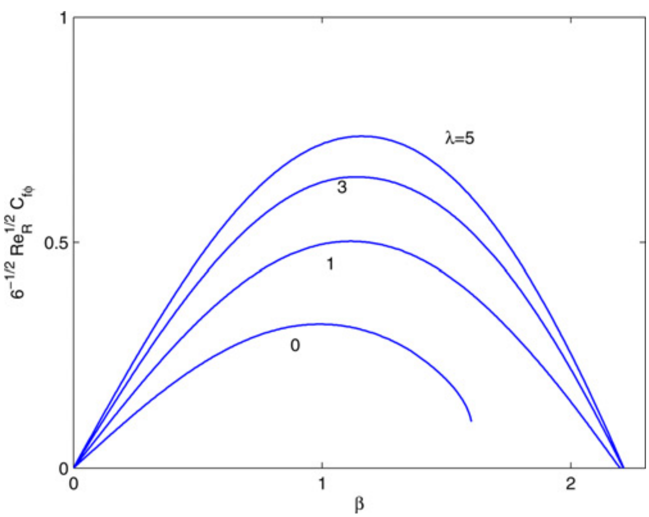

Figure 7.
Effect of λ on the skin friction coefficient in the rotating direction for $s_o = 0.1$ and $Pr = 0.7$

shifts downstream as λ increases. On the other hand, the Nusselt number ($Re_R^{-1/2}Nu$) attains its maximum value near the forward stagnation point and the minimum value away from the forward stagnation point. Further, it is to be noted that as the temperature profiles in Figure 5 reflects the opposite trend of heat transfer from fluids to the wall due to the thermal stratification of the medium, Figure 8 justifies the negative values of heat transfer rate after certain increase in the values of the angle β, i.e. above certain height h.

The effect of Prandtl number Pr on the skin friction and heat transfer coefficients $(Re_R^{1/2}C_{fx}, Re_R^{1/2}C_{f\phi}, Re_R^{-1/2}Nu)$ when $\lambda = 1, s_o = 0.1$ for various values of Pr

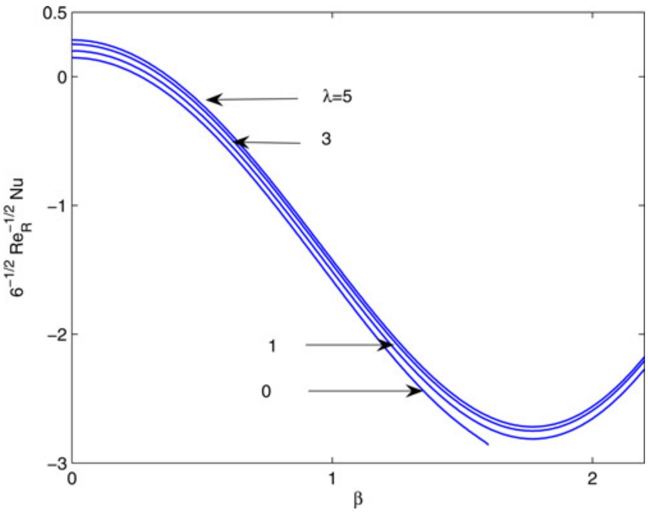

Figure 8.
Effect of λ on the Nusselt number for $s_o = 0.1$ and $Pr = 0.7$

($Pr = 0.7$, 7 and 10) shows that the momentum boundary layer grows with Pr and the thermal boundary layer thickness reduces. Hence, the skin friction coefficients decrease with increasing Pr, but the Nusselt number increases with the increase of Pr. Similar characteristic features for the effect of Pr on velocity and temperature profiles (f', s, θ) are presented in Figures 6-8 and the effects of Pr on $(Re_R^{1/2}C_{fx}, Re_R^{1/2}C_{f\phi}, Re_R^{-1/2}Nu)$ are not presented here to brief the manuscript.

Figure 9 depicts the effect of the stratification parameter s_o on the skin friction and heat transfer coefficients $(Re_R^{1/2}C_{fx}, Re_R^{1/2}C_{f\phi}, Re_R^{-1/2}Nu)$ for $\lambda = 1$ and $Pr = 0.7$. The thermal stratification parameter s_o has a significant influence on the Nusselt Number, but its effect on the skin friction coefficients is rather small. Hence, the effect on the skin friction coefficients is not shown here. As the thermal stratification parameter s_o

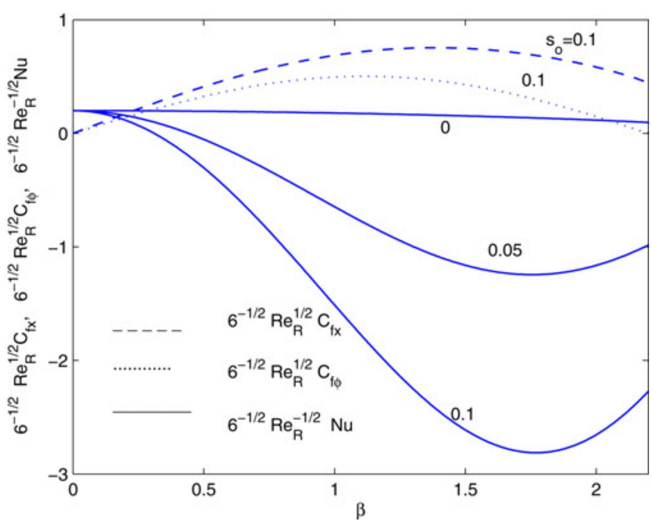

Figure 9.
Effect of s_o on the skin friction coefficients and the Nusselt number for $\lambda = 1$ and $Pr = 0.7$

increases, the non-dimensional temperature at wall decreases due to the decrease in the temperature difference between the wall and the fluid. The effect of stratification as explained earlier on temperature profiles (see Figure 5) that above certain height, heat transfer at wall occurs in opposite direction, that is, from fluids to wall, Figure 9 also indicates that the heat transfer rate at wall becomes negative after a certain increase in the angle β (i.e. above certain height h) and the effect is significant due to further increase in the value of stratification parameter s_o. The reason for the strong dependence of the heat transfer and the weak dependence of the skin friction coefficients on the thermal stratification parameter s_o is that s_o occurs explicitly in the energy equation and corresponding boundary conditions, but its effect on the skin friction coefficients is indirect (see equations (7)-(9) and (11)).

5. Conclusions

The skin friction and the heat transfer coefficients are strongly affected by the buoyancy force in the presence of thermally stratified medium. Moreover, the present study clearly points out the strong and weak dependence of buoyancy force on low ($Pr = 0.7$, air) and high ($Pr = 7$, water) Prandtl number fluids, respectively. The ambient thermal stratification parameter significantly influences the heat transfer rate but its effect on the skin friction coefficients is rather minimum. Furthermore, above certain height the heat transfer at wall reverses its direction from fluids to wall due to the presence of thermal stratification parameter. Our results are compared with previous investigators and found to be in good agreement. The present study also signifies that the heat transfer can be controlled by appropriate choice of the buoyancy force and the stratification level of the medium.

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Corresponding author
R. Ravindran can be contacted at: Ravindran.Ramalingam@wits.ac.za